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Covariant Ether Theories and Special Relativity

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Abstract

The present paper defines "covariant ether theories" as all space-time theories that are alternative to special relativity theory (SRT), but nevertheless satisfy the general relativity principle. A general analysis of the properties of admissible space-time transformations shows that the number of such "covariant ether theories" is infinite, and that all these theories are indistinguishable from SRT with all experiments in space-time physics reported to date. Some new theoretical and experimental approaches for unambiguous verification of SRT are discussed. A class of phenomena for which SRT and covariant ether theories (CET's) give different predictions is identified. It has been concluded, that under modern development of experimental technique a crucial choice between SRT and CET's can be made by means of Mössbauer spectroscopy.

1. Introduction

Modern physics accepts two relativity principles: the special Einstein's relativity principle (ERP) asserting that fundamental physical equations do not change (they are form-invariant) under transformations between inertial reference frames in an empty space, and the general relativity principle (GRP) stating that fundamental physical equations do not change their form (they are covariant) under transformations between any frames of references. (Strictly speaking, the GRP requires a covariance of physical equations with respect to "admissible" space-time transformations, which keep the requirements $g_{00} > 0$, $g_{\alpha\beta}\mathrm{d}x^a\mathrm{d}x^\beta < 0$; $\mathbf{g}$ is the metric tensor, and $\alpha, \beta = 1 \ldots 3$). The GRP is one of the deepest principles of physics and it means that any phenomenon can be described from any reference frame [1]. Thus, it does not require any experimental test. The GRP constitutes one of the corner stones of modern knowledge, and we strongly believe in its validity.

During a long time there was a widely spread opinion that ERP is a direct consequence of GRP in case of inertial motion in an empty space. If it actually was so, there would be no meaning to test ERP experimentally. Indeed, in such a case an experimental test of ERP would mean simultaneously a test of GRP, which seems to be senseless. However, ERP is not, in general, a consequence of GRP; it represents an independent physical assumption. Only in case of pseudo-Euclidean geometry of an empty space-time (that ERP requires) we get a form-invariance of physical equations with respect to the Lorentz transforms as a special inference of the covariance principle. At the same time, from a viewpoint of formal logic we are free to suppose a curvilinear geometry for an empty space-time in arbitrary inertial reference frame, with "admissible" transformations between different inertial frames. Neglecting the question about the compatibility of such a theory with the available experimental facts, one may nevertheless state that this theory would be in contradiction with ERP, but in agreement with GRP. Moreover, recent decades have witnessed the development of ether theories of such a kind, and they turned out to be indistinguishable from SRT in terms of experiments gathered up to date [1–15]. On the other hand, for many years such covariant ether theories (CETs) were often considered as pure mathematical games, even as "ether formulations of relativity" [8], which do not predict really observable non-relativistic phenomena.

For a better analysis of CET's, the present paper develops a new approach based on the difference between "physical" and "measured" values (and their transformation rules) in a hypothetical curvilinear geometry of empty space-time. This approach identifies a class of phenomena for which SRT and covariant ether theories give unambiguously different predictions, and suggests new experiments for qualitatively new tests of SRT.

2. Geometries of empty space-time

It is known that SRT establishes a pseudo-Euclidean geometry for empty physical space-time. Thus, any alternative theories of empty space-time should assume flat curvilinear geometry in arbitrary inertial reference frames. This section gives some remarks about pseudo-Euclidean and alternative geometries of empty space-time.

In the analysis of pseudo-Euclidean and curvilinear geometries of space-time, there is an essential methodological feature that has to be taken into account. Although this feature was stressed many years ago by Reichenbach (e.g., [16]), present ether theories do not take it into account explicitly.

It may be natural to believe that in any inertial reference frame we are able to construct a method for measurement of space and time intervals such that the result of measurement directly gives the physical magnitude of the corresponding interval. But strictly speaking, this is a property exclusive to pseudo-Euclidean geometry. Only in this kind of geometry can we omit a distinction between measured and physical space-time four-vectors [11,13,14]. That is, in general we have measured x_{ex} and physical x_{ph} four-vectors, and only in pseudo-Euclidean geometry do we have

$$x_{\mathrm{ph}} = x_{\mathrm{ex}} = x_{\mathrm{L}},$$

where x_{L} is the Minkowskian four-vector, subjected to the Lorentz transformation $\mathbf{L}$ between two arbitrary inertial

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frames K and K′:

$$x_{\mathrm{L}i} = L_{ij} x_{\mathrm{L}}^{\prime j}, \qquad i, j = 0 \ldots 3. \tag{1}$$

The essential property of any hypothetical curvilinear geometry of empty space-time is the difference between measured and physical space-time four-vectors in arbitrary inertial reference frames: $x_{\mathrm{ph}} \neq x_{\mathrm{ex}}$. The necessity to distinguish the measured and physical four-vectors can be easily demonstrated with the Fitzgerald–Lorentz contraction hypothesis, which was first invoked to explain the null result of the Michelson–Morley experiment. According to this hypothesis, if a rod initially at rest in the hypothetical "absolute" frame has the length l, then under motion at a constant "absolute" speed v along its axis, the length of the rod becomes $l\sqrt{1 - v^2/c^2}$. However, due to proportional contraction of the unit scale in an attached inertial reference frame, an experimenter in this frame measures the same length l as in the case $v = 0$: Fitzgerald-Lorentz contraction is not observable. Thus, we see that the length of the rod in physical space-time is $l_{\mathrm{ph}} = l\sqrt{1 - v^2/c^2}$, while the measured length is equal to $l_{\mathrm{ex}} = l$, and $l_{\mathrm{ph}} \neq l_{\mathrm{ex}}$.

One can easy demonstrate that the same conclusion is valid for time intervals in any curvilinear geometry of an empty space-time: $t_{\mathrm{ph}} \neq t_{\mathrm{ex}}$. Thus, the four-vectors in physical space-time (hereinafter 'physical four-vectors') are not in general equal to the four-vectors, whose components are constituted from corresponding experimentally measured space and time intervals (hereinafter "measured four-vectors"). Hence, in any alternative to SRT theory we have to separately derive the transformation rules for both physical and measured four-vectors.

Further, among curvilinear geometries, where we develop a space-time ether theory, a physical meaning can be prescribed only to geometries that maintain space-time homogeneity, called "oblique-angled geometries". Some essential properties of these geometries are considered in the next section.

3. From pseudo-Euclidean to oblique-angled geometry

Now let us formally construct an oblique-angled geometry by means of a transformation **B** from pseudo-Euclidean geometry:

$$(x_{\mathrm{ph}}^{\mathrm{N}})_i = B_{ij} (x_{\mathrm{ph}}^{\mathrm{E}})^j, \tag{2}$$

where the coefficients B_{ij} are some constant values and the upper superscripts E and N signify the pseudo-Euclidean and new oblique-angled geometry, respectively.

As shown above, in a non-Euclidean geometry the physical $x_{\mathrm{ph}}^{\mathrm{N}}$ and measured $x_{\mathrm{ex}}^{\mathrm{N}}$ space-time four-vectors are not in general equal to each other. Hence, a corresponding transformation from pseudo-Euclidean to oblique-angled geometry for x_{ex} should be written as

$$(x_{\mathrm{ex}}^{\mathrm{N}})_i = C_{ij} (x_{\mathrm{ex}}^{\mathrm{E}})^j, \tag{3}$$

where, in general, the matrix **C** differs from the matrix **B** in (2), but obviously depends on **B**.

In this connection one may ask the following question: what should be the form of matrix **B** in order to provide the equality $\mathbf{C} = \mathbf{E}$, where **E** is the 4×4 identity matrix? In this particular case we will have $(x_{\mathrm{ex}}^{\mathrm{N}})_i = (x_{\mathrm{ex}}^{\mathrm{E}})_i$, and an experimenter in the oblique-angled geometry will fail to detect a deflection of his geometry from pseudo-Euclidean.

In order to answer this question, let us introduce the conventional measuring methods in space-time: the use of a "unit scale" for measurement of length, the use of "standard clocks" for measurement of time, and Einstein's method for synchronization of distant clocks. Then the magnitude of the measured length $(x_{\mathrm{ex}}^{\mathrm{N}})^\alpha$ in physical space-time $x_{\mathrm{ph}}^{\mathrm{N}}$ is equal to the ratio $((x_{\mathrm{ph}}^{\mathrm{N}})^\alpha/(x_{\mathrm{phu}}^{\mathrm{N}})^\alpha)$, where $(x_{\mathrm{phu}}^{\mathrm{N}})^\alpha$ is the unit scale in the oblique-angled space-time (hereinafter the Greek subscripts correspond to three-dimensional space, $\alpha = 1, 2, 3$).

Now let us write a relationship between space components of $x_{\mathrm{ph}}^{\mathrm{N}}$ and four-vector $x_{\mathrm{ph}}^{\mathrm{E}}$:

$$(x_{\mathrm{ph}}^{\mathrm{N}})_\alpha = B_{\alpha j} (x_{\mathrm{ph}}^{\mathrm{E}})^j = B_{\alpha 0} (x_{\mathrm{ph}}^{\mathrm{E}})^0 + B_{\alpha\beta} (x_{\mathrm{ph}}^{\mathrm{E}})^\beta. \tag{4}$$

For unit scale $(x_{\mathrm{phu}}^{\mathrm{N}})_\alpha$ in physical space-time we can write the similar relation:

$$(x_{\mathrm{phu}}^{\mathrm{N}})_\alpha = B_{\alpha 0} (x_{\mathrm{ph}}^{\mathrm{E}})^0 + B_{\alpha\beta} (x_{\mathrm{phu}}^{\mathrm{E}})^\beta, \tag{5}$$

where $(x_{\mathrm{phu}}^{\mathrm{E}})^\alpha$ is the corresponding unit scale in Euclidean space. Dividing (4) by (5), one obtains:

$$\frac{(x_{\mathrm{ph}}^{\mathrm{N}})_\alpha}{(x_{\mathrm{phu}}^{\mathrm{N}})_\alpha} = \frac{B_{\alpha 0} (x_{\mathrm{ph}}^{\mathrm{E}})^0 + B_{\alpha\gamma} (x_{\mathrm{ph}}^{\mathrm{E}})^\gamma}{B_{\alpha 0} (x_{\mathrm{ph}}^{\mathrm{E}})^0 + B_{\alpha\beta} (x_{\mathrm{phu}}^{\mathrm{E}})^\beta}. \tag{6}$$

This expression allows further transformation in the particular case $B_{\alpha 0} = 0$, taking account of the obvious equality for Euclidean space

$$(x_{\mathrm{ph}}^{\mathrm{E}})^\beta/(x_{\mathrm{ph}}^{\mathrm{E}})^\alpha = (x_{\mathrm{phu}}^{\mathrm{E}})^\beta/(x_{\mathrm{phu}}^{\mathrm{E}})^\alpha.$$

Hence,

$$\begin{aligned}
\frac{(x_{\mathrm{ph}}^{\mathrm{N}})_\alpha}{(x_{\mathrm{phu}}^{\mathrm{N}})_\alpha} &= \frac{B_{\alpha\gamma} (x_{\mathrm{ph}}^{\mathrm{E}})^\gamma}{(x_{\mathrm{phu}}^{\mathrm{E}})^\alpha \left[B_{\alpha\alpha} + \left(1/(x_{\mathrm{phu}}^{\mathrm{E}})^\alpha\right) \sum_{\beta \neq \alpha} B_{\alpha\beta} (x_{\mathrm{phu}}^{\mathrm{E}})^\beta \right]} \\
&= \frac{B_{\alpha\gamma} (x_{\mathrm{ph}}^{\mathrm{E}})^\gamma}{(x_{\mathrm{phu}}^{\mathrm{E}})^\alpha \left[B_{\alpha\alpha} + \left(1/(x_{\mathrm{ph}}^{\mathrm{E}})^\alpha\right) \sum_{\beta \neq \alpha} B_{\alpha\beta} (x_{\mathrm{ph}}^{\mathrm{E}})^\beta \right]} \\
&= \frac{(x_{\mathrm{ph}}^{\mathrm{E}})_\alpha (B_{\alpha\gamma} (x_{\mathrm{ph}}^{\mathrm{E}})^\gamma)}{(x_{\mathrm{phu}}^{\mathrm{E}})_\alpha (B_{\alpha\beta} (x_{\mathrm{ph}}^{\mathrm{E}})^\beta)} = \frac{(x_{\mathrm{ph}}^{\mathrm{E}})_\alpha}{(x_{\mathrm{phu}}^{\mathrm{E}})_\alpha}.
\end{aligned} \tag{7}$$

Equality (7) means that the measured magnitudes $(x_{\mathrm{ex}}^{\mathrm{N}})_\alpha$ in $x_{\mathrm{ph}}^{\mathrm{N}}$ coordinates coincide with the measured magnitudes $(x_{\mathrm{ex}}^{\mathrm{E}})_\alpha$ in $x_{\mathrm{ph}}^{\mathrm{E}} \equiv x_{\mathrm{L}}$ coordinates, i.e.,

$$(x_{\mathrm{ex}}^{\mathrm{N}})_\alpha = (x_{\mathrm{ex}}^{\mathrm{E}})_\alpha = (x_{\mathrm{L}})_\alpha \tag{8}$$

under the condition $B_{\alpha 0} = 0$ adopted for the transformation (2). Physically, this result signifies that the distortion of the length x^α being induced by the transformation from pseudo-Euclidean to oblique-angled geometry is not detectable experimentally under $B_{\alpha 0} = 0$ because of proportional distortion of the "unit scale" x_{u}^α.

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Further, let us write a relationship between the time components of the four-vectors $x^{\mathrm{N}}_{\mathrm{ph}}$ and $x^{\mathrm{E}}_{\mathrm{ph}}$:

$$(x^{\mathrm{N}}_{\mathrm{ph}})_0 = B_{00}(x^{\mathrm{E}}_{\mathrm{ph}})^0 + B_{0\alpha}(x^{\mathrm{E}}_{\mathrm{ph}})^\alpha. \tag{9}$$

For two events at a fixed spatial point ($(x^{\mathrm{E}}_{\mathrm{ph}})^\alpha = 0$)

$$(x^{\mathrm{N}}_{\mathrm{ph}})_0 = B_{00}(x^{\mathrm{E}}_{\mathrm{ph}})^0. \tag{10}$$

Hence, the coefficient B_{00} describes the change of clock rate at a fixed spatial point under the transformation from pseudo-Euclidean to oblique-angled geometry. Such a change takes place for both a standard clock and a physical time interval. Therefore, the measured time interval at a fixed spatial point is

$$(x^{\mathrm{N}}_{\mathrm{ex}})_0 = \frac{1}{B_{00}}(x^{\mathrm{N}}_{\mathrm{ph}})_0. \tag{11}$$

For time intervals at two different spatial points separated by the distance $(x^{\mathrm{N}}_{\mathrm{ph}})_\alpha$, one should write

$$(x^{\mathrm{N}}_{\mathrm{ex}})_0 = \frac{1}{B_{00}}\Big[(x^{\mathrm{N}}_{\mathrm{ph}})_0 + \Delta(x^{\mathrm{N}}_{\mathrm{ph}})_0\Big], \tag{12}$$

where $\Delta(x^{\mathrm{N}}_{\mathrm{ph}})_0$ is the error of synchronization of clocks separated by the distance $(x^{\mathrm{N}}_{\mathrm{ph}})_\alpha$ in oblique-angled space-time. The value of $\Delta(x^{\mathrm{N}}_{\mathrm{ph}})_0$ can be found from the equality

$$(x^{\mathrm{N}}_{\mathrm{ph}})_{02} = (x^{\mathrm{N}}_{\mathrm{ph}})_{01}/2 \tag{13}$$

(Einstein's method of clocks synchronization), where $(x^{\mathrm{N}}_{\mathrm{ph}})_{01}$ is the time for light propagation from the first clock Cl_1 (at the origin of coordinates) to the second clock Cl_2 (at the point $(x^{\mathrm{N}}_{\mathrm{ph}})_\alpha$) and back according to Cl_1, while $(x^{\mathrm{N}}_{\mathrm{ph}})_{02}$ is the indication of Cl_2 at the moment of arrival of the light pulse. In oblique-angled geometry the propagation time of light from Cl_1 to Cl_2 $(x^{\mathrm{N}}_{\mathrm{ph}})_{0+}$ is not equal, in general, to the propagation time in the reverse direction $(x^{\mathrm{N}}_{\mathrm{ph}})_{0-}$. Hence, an implementation of the equality (13) is possible only in the case where the readings of both clocks at the initial moment of time differ by the value $\Delta(x^{\mathrm{N}}_{\mathrm{ph}})_0$, and

$$(x^{\mathrm{N}}_{\mathrm{ph}})_{01} = \frac{1}{B_{00}}\Big[(x^{\mathrm{N}}_{\mathrm{ph}})_{0+} + (x^{\mathrm{N}}_{\mathrm{ph}})_{0-}\Big],$$

$$(x^{\mathrm{N}}_{\mathrm{ph}})_{02} = \frac{1}{B_{00}}\Big[(x^{\mathrm{N}}_{\mathrm{ph}})_{0+} + \Delta(x^{\mathrm{N}}_{\mathrm{ph}})_0\Big]. \tag{14}$$

Hence, with account of Eq. (13) we obtain:

$$\Delta(x^{\mathrm{N}}_{\mathrm{ph}})_0 = \frac{1}{2}\Big[(x^{\mathrm{N}}_{\mathrm{ph}})_{0-} - (x^{\mathrm{N}}_{\mathrm{ph}})_{0+}\Big]. \tag{15}$$

Expressions for $(x^{\mathrm{N}}_{\mathrm{ph}})_{0+}$ and $(x^{\mathrm{N}}_{\mathrm{ph}})_{0-}$ can be found from Eq. (2):

$$(x^{\mathrm{N}}_{\mathrm{ph}})_{0+} = B_{00}(x^{\mathrm{E}}_{\mathrm{ph}})^0 + B_{0\alpha}(x^{\mathrm{E}}_{\mathrm{ph}})^\alpha,$$

$$(x^{\mathrm{N}}_{\mathrm{ph}})_{0-} = B_{00}(x^{\mathrm{E}}_{\mathrm{ph}})^0 - B_{0\alpha}(x^{\mathrm{E}}_{\mathrm{ph}})^\alpha. \tag{16}$$

Substituting Eq. (16) into Eq. (15), one gets:

$$\Delta(x^{\mathrm{N}}_{\mathrm{ph}})_0 = -B_{0\alpha}(x^{\mathrm{E}}_{\mathrm{ph}})^\alpha. \tag{17}$$

Further substitution of Eqs. (17) and (9) into Eq. (12) gives:

$$(x^{\mathrm{N}}_{\mathrm{ex}})_0 = (x^{\mathrm{E}}_{\mathrm{ph}})_0 = (x^{\mathrm{E}}_{\mathrm{ex}})_0 = (x_{\mathrm{L}})_0. \tag{18}$$

Therefore, we conclude that for any admissible transformation **B**, an experimenter will not detect a deflection of his geometry from pseudo-Euclidean under measurement of the time intervals. Besides, under the condition $B_{\alpha 0} = 0$ the same conclusion is additionally valid for the measurement of length, so a deflection of geometry of physical space-time from pseudo-Euclidean is not detectable experimentally. On the other hand, the transformation **B** under $B_{\alpha 0} = 0$ belongs to a class of transformations $x'^0 = x'^0(x^i)$, $x'^\alpha = x'^\alpha(x^\beta)$ acting within fixed reference frames. Thus, we have proved the following general theorem:

> Under any admissible transformation **B** from pseudo-Euclidean to oblique-angled geometry within a fixed frame of reference ($B_{\alpha 0} = 0$) a deflection of geometry from pseudo-Euclidean is not experimentally observable, and the equalities (8) and (18) obtained above can be written simultaneously as
>
> $$(x^{\mathrm{N}}_{\mathrm{ex}})^i = (x^{\mathrm{E}}_{\mathrm{ex}})^i = (x_{\mathrm{L}})^i. \tag{19}$$

This theorem will play a key role in the physical interpretation of covariant ether theories considered in the next section.

4. Space-time transformations in covariant ether theories

The equality (19) obtained above means that for any hypothetical oblique-angled geometry of an empty physical space-time, related with the pseudo-Euclidean one by means of transformation **B** under $B_{\alpha 0} = 0$, the measured space-time four-vectors in properly constructed inertial reference frames are subject to Lorentz transformations. In order to derive physical inferences from this result, it is necessary to find a possible physical meaning for such **B**-transformations. This can be done in the following way:

Let us consider a class of space-time theories of inertial motion in an empty space satisfying the following general principles: space-time homogeneity; space isotropy; causality principle; general relativity principle (GRP). In these theories, a general transformation between two arbitrary inertial reference frames K and K′ in physical space-time can be written as

$$x_{\mathrm{ph}i} = A_{ij}x'^j_{\mathrm{ph}}, \tag{20}$$

where allowable transformations **A** in (20) are linear due to the space-time homogeneity and constitute a ten-parametrical Lee group due to adoption of the GRP [17]. For simplicity we further omit trivial translations and rotations of space, considering three-parametrical transformations $\mathbf{A}(\boldsymbol{v})$, where $\boldsymbol{v}$ is the relative velocity of two arbitrary inertial frames. In addition, the GRP also requires the validity of a reciprocity principle [18]: the

mutual velocities of two inertial reference frames should differ only by sign; i.e.,

$$\mathbf{A}^{-1}(\boldsymbol{v}) = \mathbf{A}(-\boldsymbol{v}). \tag{21}$$

In its turn, the equality (21) and causality principles ensure that [18]

$$\det\mathbf{A} = 1. \tag{22}$$

Thus, the transformations **A** are "special orthogonal".

Further, we do not in general suppose that the geometry of physical space-time has to be pseudo-Euclidean in any inertial reference frame. At the same time, due to the isotropy of physical space, there exists an inertial reference frame K_0 such that the speed of light is isotropic and equal to c; i.e., its geometry should be pseudo-Euclidean under any particular choice of admissible **A**-transformation. In order to specify this requirement, the existence of at least one K_0 frame with pseudo-Euclidean geometry for any admissible transformation **A**, we can formally introduce into consideration the Minkowskian coordinates x_L and demand the equality (23) for the frame K_0 :

$$x'_{\mathrm{ph}i} \doteq x'_{\mathrm{L}i} \tag{23}$$

(hereinafter the primed four-vectors belong to K_0), and in general case $\mathbf{A} \neq \mathbf{L}$ equality (23) is valid only for this ("absolute") frame. One can additionally show that Eq. (23) simultaneously ensures that the x_{ph} coordinates are "admissible" wherein the known relationships between components of the metric tensor **g** take place [11], and the velocity of light is finite [13].

Using Eqs. (1), (20), (23), one can find a relation between x_{ph} and x_{L} in an inertial frame K moving at the constant velocity $\boldsymbol{v}$ in K_0 :

$$x_{\mathrm{ph}i} = B_{ij}(\boldsymbol{v})x^j_{\mathrm{L}}, \tag{24}$$

where the introduced matrix **B** is determined by the relationship

$$B_{ij}(\boldsymbol{v}) = A^k_i(\boldsymbol{v})L^{-1}_{kj}(\boldsymbol{v}). \tag{25}$$

At the same time, the four-vector x_L, defined in the frame K, does not depend on the velocity $\boldsymbol{v}$ of K in K_0. Therefore, x_L should be considered as a constant in Eq. (24). Its constant magnitude is determined for the particular case $v = 0$, where the matrices **A**, **L** are equal to the unit matrix, and

$$x^j_{\mathrm{L}} = x^j_{\mathrm{ph}}(v = 0). \tag{26}$$

Substituting Eq. (26) into Eq. (24), one gets the dependence of the physical space-time four-vectors x_{ph} on the "absolute" velocity $\boldsymbol{v}$, which will be useful in further consideration:

$$x_{\mathrm{ph}i}(\boldsymbol{v}) = B_{ij}(\boldsymbol{v})x^j_{\mathrm{ph}}(v = 0). \tag{27}$$

Further, using a known form of the matrix **L** (see, for instance [19]) one obtains from Eq. (25) a relationship between the matrices **B** and **A**:

$$B_{00} = \frac{\gamma}{A_{00}}; \tag{28a}$$

$$B_{\alpha 0} = 0; \tag{28b}$$

$$B_{0\alpha} = A_{0\alpha} + A_{00} \times \left[\frac{v_\alpha}{c^2}\gamma + (\frac{1}{A^2_{00}} - 1)\frac{v_\alpha}{v^2}(\gamma - 1)\right]; \tag{28c}$$

$$B_{\alpha\beta} = A_{\alpha\beta} + A_{\alpha 0}\frac{v_\beta}{v^2}\left(1 - \frac{1}{\gamma}\right), \tag{28d}$$

where $\gamma = 1/\sqrt{1 - (v^2/c^2)}$. (The derivation of Eqs. (28a) and (28b) takes into account Eq. (21)).

Paying attention to Eq. (28b), we see that the coefficient $B_{\alpha 0}$ is always equal to zero for any admissible transformation **A**; this is a necessary and sufficient condition for implementation of the equality (19). Hence, we conclude that for any space-time theory of inertial motion satisfying to general symmetries of space-time and the general relativity principle, the space-time four-vectors measured in experiments always obey the Lorentz transformation **L** in Galilean inertial reference frames (corresponding to Cartesian frames in three-dimensional space). From here follows the fact that Lorentz transformation for measured space and time intervals formally gives no information about the geometry of physical space-time, and does not generally reject ether theories with oblique-angled space-time.

Let us show that the number of such admissible theories is infinite. Indeed, in the general case the number of coefficients in the matrix **A** is equal to 16. In the simplest one-dimensional case this number is reduced to 4. Indeed, in the one-dimensional case the transformation (20) is written as

$$\begin{aligned} (x_{\mathrm{ph}})_0 &= A_{00}(x'_{\mathrm{ph}})^0 + A_{01}(x'_{\mathrm{ph}})^1, \\ (x_{\mathrm{ph}})_1 &= A_{10}(x'_{\mathrm{ph}}t)^0 + A_{11}(x'_{\mathrm{ph}})^1. \end{aligned} \tag{29}$$

However, in such a case we are able to get only three equations for coefficients of **A**. One can show that the reciprocity principle (Eq. (21)) provides the equality:

$$A_{11} = A_{00}. \tag{30}$$

The relationship of A_{10} with A_{00} can be found from the requirement:

$$\left[(x'_{\mathrm{ph}})^1/(x'_{\mathrm{ph}})^0\right]_{(x_{\mathrm{ph}})_1=0} = v.$$

The last requirement and Eq. (29) lead to the equality:

$$A_{10} = -vA_{00}. \tag{31}$$

Further, Eqs. (22), (30) and (31) allow expressing A_{01} as:

$$A_{01} = \frac{1 - (A_{00})^2}{vA_{00}}. \tag{32}$$

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Thus, the three obtained Eqs. (30)–(32) establish a relationship between the coefficient A_{00} and all other coefficients of the matrix $\mathbf{A}$ in one-dimensional case. However, there are no restrictions on the concrete choice of A_{00}. This means the number of space-time theories where the measured space-time four-vectors are subjected to the Lorentz transforms is infinite. That is why all the experiments for verification of Lorentz transforms (beginning with the Michelson–Morley experiment and finishing with the modern Champeney experiment [20]) find an infinite number of alternative explanations of their results.

We conditionally name the acceptable theories "covariant ether theories". The exclusive place of SRT among all such covariant ether theories is defined by the fact that it directly asserts the equality of measured and physical space-time four-vectors, i.e., the equality of the matrices $\mathbf{A}$ and $\mathbf{L}$. On the other hand, this assertion follows from nothing: in general, a question about coincidence of "measured" and "physical" values even under optimal measurements is one of the fundamental questions to Nature. That is why it is especially interesting to analyze a negative answer to this question. One can easily see that a negative answer means that $\mathbf{A} \neq \mathbf{L}$. Under $\mathbf{A} \neq \mathbf{L}$, the geometry of physical space-time is oblique-angled, and we must distinguish space-time transformations for physical and measured four-vectors:

$$x_{\mathrm{ph}i} = A_{ij} x'^{j}_{\mathrm{ph}}, \tag{33}$$

$$x_{\mathrm{ex}i} = L_{ij} x'^{j}_{\mathrm{ex}}, \tag{34}$$

where the primed four-vectors, as before, belong to the "absolute" frame K_0. This means that these transformations do not yet solve the main kinematical problem (determination of space-time transformations between two arbitrary inertial frames): it acts only in the special case defined by the equality (23). In order to find a transformation between two such arbitrary inertial frames K and K″, we should write

$$x_{\mathrm{ex}i} = L_{ij}(\boldsymbol{v}_1) x'^{j}_{\mathrm{ex}}; \qquad x''_{\mathrm{ex}i} = L_{ij}(\boldsymbol{v}_2) x'^{j}_{\mathrm{ex}}, \tag{35}$$

$$x_{\mathrm{ph}i} = A_{ij}(\boldsymbol{v}_1) x'^{j}_{\mathrm{ph}}; \qquad x_{\mathrm{ph}i} = A_{ij}(\boldsymbol{v}_2) x'^{j}_{\mathrm{ph}}, \tag{36}$$

where $\boldsymbol{v}_1, \boldsymbol{v}_2$ are the "absolute" velocities of the frames K and K″, respectively. Excluding four-vectors x'^{j}_{ex} from Eqs. (35), and x'^{j}_{ph} from Eq. (36), we obtain general transformations for measured and physical space-time four-vectors in two arbitrary inertial frames:

$$x_{\mathrm{ex}i} = L_{ij}(\boldsymbol{v}_1)[L^{-1}(\boldsymbol{v}_2)]^{jk} x''_{\mathrm{ex}k}, \tag{37}$$

$$x_{\mathrm{ph}i} = A_{ij}(\boldsymbol{v}_1)[A^{-1}(\boldsymbol{v}_2)]^{jk} x''_{\mathrm{ph}k}, \tag{38}$$

where the matrix $\mathbf{A}$ can be taken in arbitrary admissible form.

Thus in contrast to SRT, under the hypothesis $\mathbf{A} \neq \mathbf{L}$, Nature does not "know" a direct relative velocity of two arbitrary inertial frames K and K″: it is always composed as a sum $\boldsymbol{v}_1 \oplus \boldsymbol{v}_2$, where $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are the corresponding velocities of K and K″ in the "absolute" frame K_0. This means, in particular, that direct rotation-free Lorentz transformation between measured space-time four-vectors in K and K″ is impossible: according to general group properties of these transformations, an additional rotation of the coordinate axes of the frames K and K″ appears at the so called Thomas–Wigner angle Ω, depending on $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. It is quite important that such a rotation occurs in measured coordinates, i.e., it can really be detected. It defines a principal possibility to experimentally distinguish the hypotheses $\mathbf{A} = \mathbf{L}$ and $\mathbf{A} \neq \mathbf{L}$.

Among admissible space-time theories that assume $\mathbf{A} \neq \mathbf{L}$, the simplest case corresponds to the choice $\mathbf{A} = \mathbf{G}$, where $\mathbf{G}$ is the matrix of Galilean transformation: $G_{ii} = 1$, $G_{\alpha 0} = -v_\alpha$, and all other $G_{ij} = 0$. Substituting the matrix $\mathbf{G}$ in place of matrix $\mathbf{A}$ in Eq. (28), one gets the following coefficients of matrix $\mathbf{B}$:

$$\begin{aligned} &B_{00} = \gamma, \qquad B_{\alpha 0} = 0, \qquad B_{0\alpha} = \frac{v_\alpha}{c^2}\gamma, \\ &B_{\alpha\beta} = \delta_{\alpha\beta}\frac{v_\alpha v_\beta}{v^2}\left(1 - \frac{1}{\gamma}\right), \end{aligned} \tag{39}$$

where $\delta_{\alpha\beta}$ is the Kronekker symbol. Further substitution of Eq. (39) into Eq. (27) allows one to determine the dependence of physical space-time four-vectors on the "absolute" velocity $\boldsymbol{v}$ of some arbitrary inertial reference frame K:

$$\boldsymbol{r}_{\mathrm{ph}}(\boldsymbol{v}) = \boldsymbol{r}_{\mathrm{ph}}(v = 0) + \frac{\boldsymbol{v}(\boldsymbol{r}_{\mathrm{ph}}(v = 0), \boldsymbol{v})}{v^2}[\sqrt{1 - (v^2/c^2)} - 1], \tag{40}$$

$$t_{\mathrm{ph}}(\boldsymbol{v}) = \frac{t_{\mathrm{ph}}(v = 0)}{\sqrt{1 - (v^2/c^2)}} + \frac{\boldsymbol{r}_{\mathrm{ph}}(v = 0)\boldsymbol{v}}{c^2\sqrt{1 - (v^2/c^2)}}. \tag{41}$$

For the considered time intervals in a fixed spatial point of the frame K ($r_{\mathrm{ph}} = 0$), we obtain the dependence of t_{ph} on $\boldsymbol{v}$ [see, Eq. (41)]:

$$t_{\mathrm{ph}}(\boldsymbol{v}) = t_{\mathrm{ph}}(v = 0)/\sqrt{1 - (v^2/c^2)}, \tag{42}$$

that means an "absolute" dilation of time by the factor $\sqrt{1 - (v^2/c^2)}$. Furthermore, one obtains from Eq. (40):.

$$\begin{aligned} &\big(\boldsymbol{r}_{\mathrm{ph}}(\boldsymbol{v}), \boldsymbol{v}\big) = \big(\boldsymbol{r}_{\mathrm{ph}}(v = 0), \boldsymbol{v}\big)\sqrt{1 - v^2/c^2}, \\ &\big[\boldsymbol{r}_{\mathrm{ph}}(\boldsymbol{v}) \times \boldsymbol{v}\big] = \big[\boldsymbol{r}_{\mathrm{ph}}(v = 0) \times \boldsymbol{v}\big], \end{aligned} \tag{43}$$

which means an "absolute" contraction of a moving scale along a vector of "absolute" velocity (Fitzgerald–Lorentz hypothesis). Finally, transformation (38) (under $\mathbf{A} = \mathbf{G}$)

$$x_{\mathrm{ph}i} = [G_{ij}(\boldsymbol{v}_1 - \boldsymbol{v}_2)]x''_{\mathrm{ph}}j$$

leads to the Galilean law of speed addition for the physical light velocity c_{ph}.

Thus, we have got a full set of Lorentz ether postulates in case $\mathbf{A} = \mathbf{G}$.[1] However, the physical space-time in the

[1]Let us recall the postulates of Lorentz ether theory in its modern form: (1) There is an "absolute" reference frame K_0, wherein the light velocity is isotropic and equal to c. (2) In an arbitrary reference frame K, moving at constant velocity $\boldsymbol{v}$ in K_0, the velocity of light is equal to $\boldsymbol{c}' = \boldsymbol{c} - \boldsymbol{v}$. (3) In this reference frame K time is dilated by $\sqrt{1 - v^2/c^2}$ times. (4) In this reference frame K a linear scale is contracted by $\sqrt{1 - v^2/c^2}$ times along the vector $\boldsymbol{v}$.

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Lorentz ether theory is not observable in an arbitrary inertial reference frame, while the measured four-vectors x_{ex} obey to the Lorentz transformations in form of (37). (This important circumstance about a difference of physical and measured four-vectors in non-Euclidean geometry of moving inertial frames was dropped by Lorentz and his successors). Therefore, we may consider the Lorentz ether theory as one of the CET's defined above, and the simplest among them. Due to this fact, the application of the Lorentz ether postulates for explanation of "null" results of all experiments searching for "ether wind speed" was always successful.

5. Experimental test of CET's

So, we have found that the difference between SRT ($\mathbf{A} = \mathbf{L}$) and CET's ($\mathbf{A} \neq \mathbf{L}$) appears on an experimental level only in successive space-time transformations. It follows from there that an instrument for measuring hypothetical "absolute" velocity must contain moving inertial parts, in order to deal with such transformations. Then a general idea of an experiment for a choice between SRT and CET's can be described with help of the diagram in Fig. 1. It shows the absolute frame K_0, laboratory frame K (moving at the constant absolute velocity $\boldsymbol{v}$) and frame K_i, attached to some moving inertial part of a measuring instrument in K.

In our laboratory we specify a velocity $\boldsymbol{u}$ of K_i in K. In such a case for the hypothesis $\mathbf{A} = \mathbf{L}$ we apply a direct rotation-free Lorentz transformation $K \to K_i$ for calculation of the indication of the measuring device. Hence, according to SRT we get a vanishing value of absolute velocity. Under the hypothesis $\mathbf{A} \neq \mathbf{L}$, Nature does not "know" a direct rotation-free Lorentz transformation between K and K_i, and "operates" with the absolute velocities of these frames $\boldsymbol{v}$ and $\boldsymbol{v} \oplus \boldsymbol{u}$. Hence, in order to calculate an indication of the measuring device, we must apply the successive transformations $K \to K_0 \to K_i$ according to Eq. (37). (A direct Lorentz transformation from K to K_i is also possible, but it will not be rotation-free). In this case the axes of the frames K and K_i are turned out at the Thomas–Wigner angle Ω, that, in principle, changes the state of the measuring instrument. Since Ω depends on the absolute velocity $\boldsymbol{v}$ of the laboratory frame K, the state of the measuring instrument will depend on $\boldsymbol{v}$, too. There is only one particular case ($\boldsymbol{v}$ is collinear to $\boldsymbol{u}$) where $\Omega = 0$, and the state of the measuring instrument has to be unchanged for any magnitude of absolute velocity of the laboratory frame. This is an unambiguous inference from the GRP. All experiments searching for "ether wind" velocity with experimental instruments containing moving inertial parts, aiming to measure non-relativistic effects under collinear $\boldsymbol{v}$ and $\boldsymbol{u}$ (see, e.g., [20–22]), in fact checked the GRP, not the Einstein relativity principle.

Thus, an experiment for qualitatively new tests of SRT must contain moving inertial part (parts) with non-collinear velocities $\boldsymbol{v}$ and $\boldsymbol{u}$, and be aimed to measure the dependence of the angle Ω on the absolute velocity of the laboratory frame. To the order of magnitude c^{-2}, and for orthogonal vectors of $\boldsymbol{v}$ and $\boldsymbol{u}$, this dependence is defined by the expression (see, e.g. [19]) $\Omega = -uv/2c^2$.

A direct measurement of this dependence in a laboratory-scale experiment is impractical. Indeed, the absolute speed v could be taken as about $10^{-3}c$ (typical velocities of Galaxy objects). The maximum value of u could be about 10^3 m/s. Hence, the angle Ω takes on the value 3×10^{-9}, i.e., well below any limit of practicability in a laboratory experiment.

An analysis of possible experimental schemes for indirect measurement of the angle Ω can be greatly simplified under numerical estimation of eventual non-relativistic effects, proceeding from their dimension. Indeed, the experiments, looking for the change of length associated with the $\Omega(\boldsymbol{v})$ dependence, give an effect in the order of magnitude $L\Omega$. Here L is some length, which is equal to about 1 m in the laboratory scale. In such a case we get $L\Omega \approx 3 \times 10^{-9}$ m, which is impossible to measure in practice. A corresponding change of time has a dimension $L\Omega/u = Lv/c^2 \approx 3$ ps, a time interval within the range of present technology, but not for mechanical parts necessarily involved. Finally, one can rearrange the experiment into a "speed experiment" looking at the term Ωu, and the latter arrangement could be further transformed into frequency measurement via the Doppler effect ($\Omega u/c$). For the last case one is looking at the term u^2v/c^3, the latter being about 10^{-14} (for $u \approx 10^3$ m/s)—a value accessible practically conveniently only by the Mössbauer effect, at least as far as a laboratory-scale experiment is concerned.

Thus, an indirect measurement of the $\Omega(\boldsymbol{v})$ dependence in a Mössbauer experiment is a real way to unambiguously check SRT. It can be accomplished in Champeney-type experiments, using the most recent methodological developments in Mössbauer spectroscopy: resonant method of registration of recoil-free radiation [13,23] or nuclear resonant scattering of synchrotron radiation [15].

6. Conclusions

1. Consideration of all hypothetical theories of empty space-time with curvilinear geometry should be based on distinguishing between physical and measured space-time four-vectors. General analysis of the properties of admissible space-time transformations shows that in

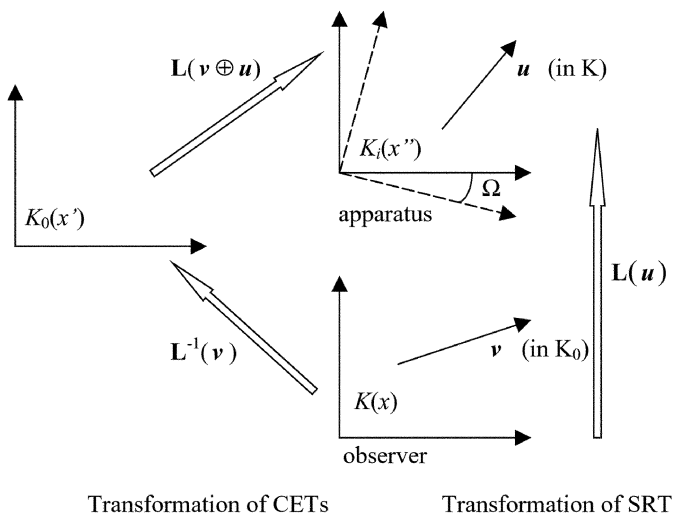

Fig. 1. General idea for an experiment to provide a new test of Special Relativity Theory.

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any theory adopting the general relativity principle and symmetries of space-time, the measured space and time intervals always obey the Lorentz transformation, regardless of a concrete choice of physical space-time transformation. The latter circumstance makes it possible to explain all known experimental results in space-time physics within an infinite number of admissible space-time theories, called "covariant ether theories".

2. SRT is unique among admissible theories of empty space-time because it directly asserts an equality between measured and physical four-vectors under optimal measurements. Adoption of such an equality defines the possibility of direct rotation-free Lorentz transformation between two arbitrary inertial frames. This is impossible in all other admissible space-time theories, and they lead to a dependence of rotation angle Ω on an "absolute" velocity $\boldsymbol{v}$ for the measured space-time co-ordinates, subjected to the transformations (37). Hence, the hypothetical dependence $\Omega(\boldsymbol{v})$ is the sole observable physical phenomenon allowing an unambiguous test of SRT.
3. The most convenient method for the measurement of $\Omega(\boldsymbol{v})$ dependence under modern development of laboratory experimental technique is Mössbauer spectroscopy: either a resonant method of registration of recoil-free radiation, or nuclear resonant scattering of synchrotron radiation in Champeney-type experiments.

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